Experimental Observation of Stochastic Resonance in a Magnetically Driven Mechanical Duffing Oscillator

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What is the Duffing Oscillator

- Duffing oscillator comes from potential function:

$$U(x) = \frac{a}{4}x^4 - \frac{b}{2}x^2$$

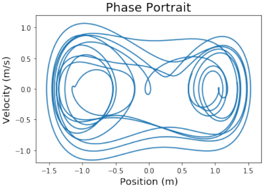
$$F(x) = -\frac{dU}{dx} = bx - ax^3$$

- Using Newton's second law and adding a damping and driving force:

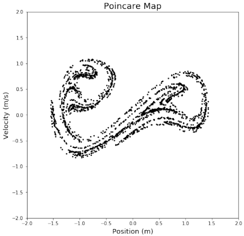
$$\ddot{x} + \delta\dot{x} + kx = F\cos(\Omega t) \qquad \ddot{x} + \delta\dot{x} + ax^3 - bx = F\cos(\Omega t)$$

Numerical Model

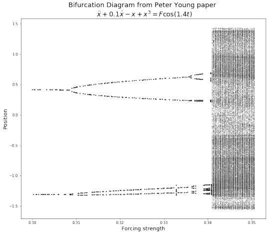
- Our numerical model simulates ODE with Runge-Kutta ODE solver. Duffing oscillator displays chaotic characteristics:



(a) Phase portrait



(b) Poincaré map



(c) Bifurcation behavior

Figure 1: Preliminary Results Presented at Kentucky Academy of Science 2019 Meeting

Stochastic Resonance

- Because Duffing oscillator is a nonlinear, bistable system, it can experience stochastic resonance (SR).
- SR is when a nonlinear system experiences resonance due to some amount of noise being added to it:

$$\ddot{x} + \delta\dot{x} + ax^3 - bx = F\cos(\Omega t) + \nu(t)$$

- SR can be characterized by appearance of forcing frequency in oscillator position:

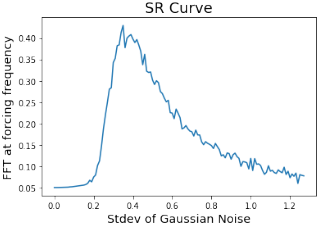


Figure 2: Sample Numerically Calculated SR curve

Why Study Stochastic Resonance

- SR shows up in many real life systems
- Two examples:

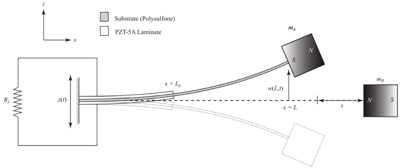


Figure 3: Energy harvesting using piezoelectric beams

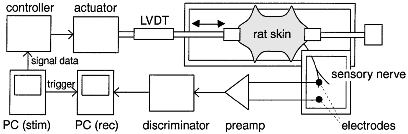


Figure 4: Controlling nerve firing in sensory tissues

Mechanical Design

- Design inspired by *Donoso, Ladera, Eur. J. Phys. 33 (2012) 1473–1486*

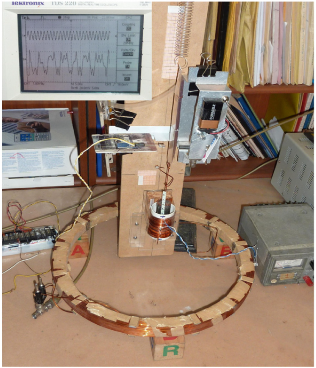


Figure 5: Design by *Donoso, Ladera*

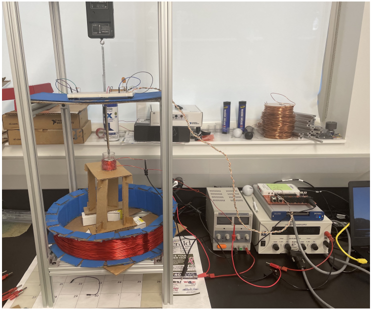


Figure 6: WKU Setup

Data Acquisition System

- National Instruments *USB-6431* connects to position measurement system, and power supply control.

Figure 7: *USB-6431* with power supplies

- LabVIEW connects to setup to synchronize acquisition and control.

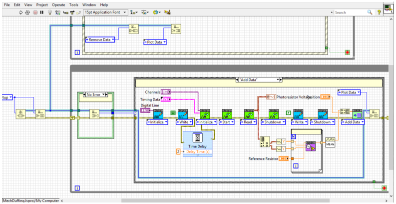


Figure 8: LabVIEW for data acquisition

- LabVIEW uses functional global variables and queued state machines to handle IO functions

Sensor Position Data

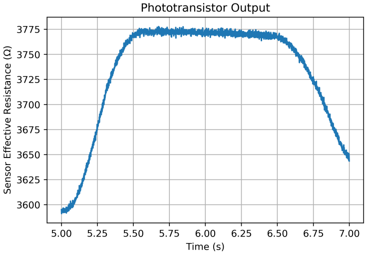


Figure 9: Position data from Phototransistor

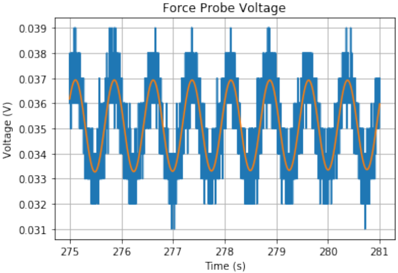


Figure 10: Position data from force probe

Duffing Position Data

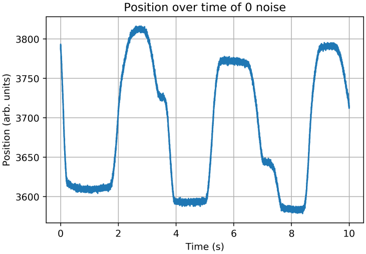


Figure 11: No Noise

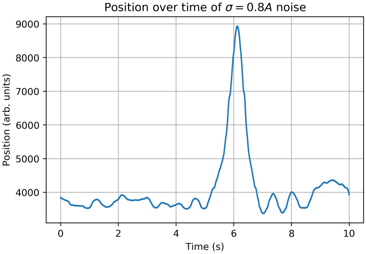


Figure 12: Noise Added

Duffing Phase Portraits

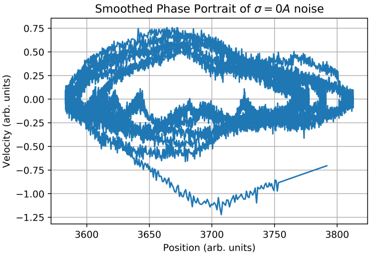


Figure 13: No Noise

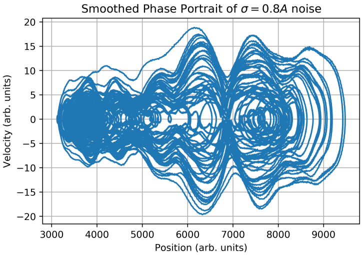


Figure 14: Noise Added

Duffing Fourier Analysis of System Response

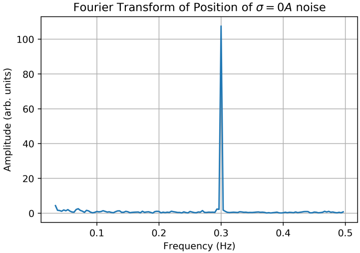


Figure 15: No Noise

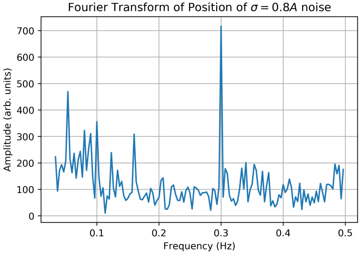


Figure 16: Noise Added

Experimental Stochastic Resonance Curve

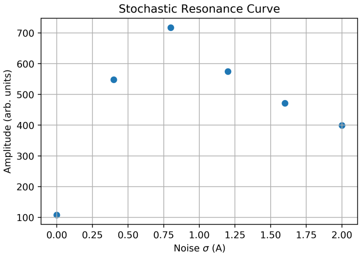


Figure 17: Experimental SR Curve

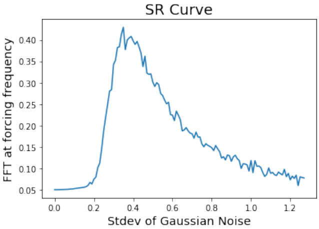


Figure 18: Sample Numerically Calculated SR Curve, Not Same System

Results

- We have experimentally measured SR in a quasi-Duffing oscillator
- SR can be measured in a variety of systems, as long as they are bistable or have a hysteresis loop